Mathematics: Analysis and Approaches SL

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Topic 1: Number and algebra

1.2 nth term of an arithmetic sequence
$$u_n = u_1 + (n-1)d$$

Sum of n terms of an arithmetic sequence
$$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$$

1.3 nth term of a geometric sequence
$$u_n = u_1 r^{n-1}$$

Sum of n terms of a finite geometric sequence
$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$$

1.8 Sum of an infinite geometric sequence
$$S_\infty = \frac{u_1}{1 - r}, \quad |r| < 1$$

1.4 Compound interest
$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$

1.5 Exponents and logarithms $(a, b > 0,\ a \neq 1)$
$$a^x = b \iff x = \log_a b$$

1.7 Laws of logarithms
$$\log_a xy = \log_a x + \log_a y$$
$$\log_a \frac{x}{y} = \log_a x - \log_a y$$
$$\log_a x^m = m \log_a x$$

Change of base
$$\log_a x = \frac{\log_b x}{\log_b a}$$

Exponential and logarithmic functions
$$a^x = \mathrm{e}^{x \ln a}; \quad \log_a a^x = x = a^{\log_a x}$$

1.9 Binomial theorem, $n \in \mathbb{N}$
$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$
$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

Topic 2: Functions

2.1 Equations of a straight line
$$y = mx + c$$
$$ax + by + d = 0$$

Gradient formula
$$y - y_1 = m(x - x_1)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

2.6 Axis of symmetry of a quadratic
$$f(x) = ax^2 + bx + c \Rightarrow x = -\frac{b}{2a}$$

2.7 Solutions of a quadratic equation
$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$$

Discriminant
$$\Delta = b^2 - 4ac$$

Topic 3: Geometry and trigonometry – prior learning

Area of a parallelogram
$$A = bh \quad (b \text{ base},\ h \text{ height})$$

Area of a triangle
$$A = \tfrac{1}{2}bh$$

Area of a trapezoid
$$A = \tfrac{1}{2}(a+b)h \quad (a, b \text{ parallel sides})$$

Area of a circle
$$A = \pi r^2$$

Circumference of a circle
$$C = 2\pi r$$

Volume of a cuboid
$$V = lwh$$

Volume of a cylinder
$$V = \pi r^2 h$$

Volume of a prism
$$V = Ah \quad (A \text{ cross-section})$$

Curved surface area of a cylinder
$$A = 2\pi rh$$

Distance between (x_1, y_1) and (x_2, y_2)
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Midpoint of (x_1, y_1) and (x_2, y_2)
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Topic 3: Geometry and trigonometry

3.1 Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2)
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Midpoint of (x_1, y_1, z_1) and (x_2, y_2, z_2)
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Volume of a right pyramid
$$V = \tfrac{1}{3}Ah \quad (A \text{ base area})$$

Volume of a right cone
$$V = \tfrac{1}{3}\pi r^2 h$$

Curved surface area of a cone
$$A = \pi rl \quad (l \text{ slant height})$$

Volume of a sphere
$$V = \tfrac{4}{3}\pi r^3$$

Surface area of a sphere
$$A = 4\pi r^2$$

3.2 Sine rule
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Area of a triangle
$$A = \tfrac{1}{2}ab\sin C$$

3.4 Length of an arc (θ in radians)
$$l = r\theta$$

Area of a sector (θ in radians)
$$A = \tfrac{1}{2}r^2\theta$$

3.5 Identity for $\tan\theta$
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

3.6 Pythagorean identity
$$\cos^2\theta + \sin^2\theta = 1$$

Double angle identities
$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

Topic 4: Statistics and probability

4.2 Interquartile range
$$\mathrm{IQR} = Q_3 - Q_1$$

4.3 Mean of a data set
$$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}, \qquad n = \sum_{i=1}^{k} f_i$$

4.5 Probability of an event A
$$\mathrm{P}(A) = \frac{n(A)}{n(U)}$$

Complementary events
$$\mathrm{P}(A) + \mathrm{P}(A') = 1$$

4.6 Combined events
$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

Mutually exclusive events
$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$$

Conditional probability
$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$

Independent events
$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$$

4.7 Expected value of a discrete random variable X
$$\mathrm{E}(X) = \sum_{i=1}^{k} x_i\,\mathrm{P}(X = x_i)$$

4.8 Binomial distribution
$$X \sim \mathrm{B}(n, p)$$

Mean and variance
$$\mathrm{E}(X) = np, \qquad \mathrm{Var}(X) = np(1-p)$$

4.12 Standardized normal variable
$$z = \frac{x - \mu}{\sigma}$$

Topic 5: Calculus

5.3 Derivative of x^n
$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

5.6 Standard derivatives
$$f(x) = \sin x \Rightarrow f'(x) = \cos x$$
$$f(x) = \cos x \Rightarrow f'(x) = -\sin x$$

e^x and $\ln x$
$$f(x) = \mathrm{e}^x \Rightarrow f'(x) = \mathrm{e}^x$$
$$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$$

Chain rule
$$y = g(u),\ u = f(x) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$$

Product rule
$$y = uv \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$$

Quotient rule
$$y = \frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

5.9 Kinematics
$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$$

Distance travelled from t_1 to t_2
$$\int_{t_1}^{t_2} |v(t)|\,\mathrm{d}t$$

Displacement from t_1 to t_2
$$\int_{t_1}^{t_2} v(t)\,\mathrm{d}t$$

5.5 Integral of x^n
$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Area between $y = f(x)$ and the x-axis, $f(x) > 0$
$$A = \int_a^b y\,\mathrm{d}x$$

5.10 Standard integrals
$$\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + C$$

5.11 Area of the region enclosed by a curve and the x-axis
$$A = \int_a^b y\,\mathrm{d}x$$